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A Vlasov solver for collective effects in particle accelerators

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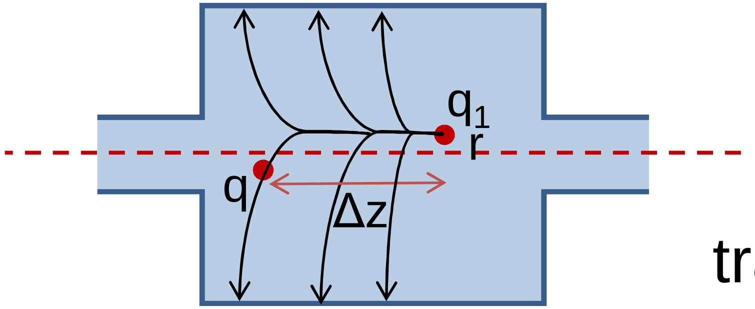
Talk outline

- Wake fields
- Beam dynamics and multiparticle codes
- Vlasov solver
- Exponential operators
- Equations and simulations for 1D and 2D magnetic bunch compressor and Storage Rings
- Conclusions

Wake fields



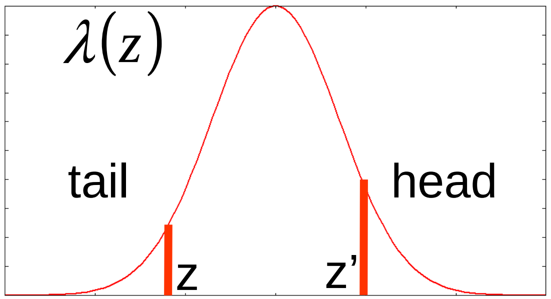
Hp: cylindrical symmetry

longitudinal $$w_{\|}(\Delta z)=\frac{1}{qq_1}\int_{Structure}F_{L\|}(\Delta z;s)ds$$

transverse

$$w_{\perp}(\Delta z,r)=\frac{1}{qq_1}\int_{Structure}F_{L\perp}(\Delta z,r;s)ds=w'_{\perp}(\Delta z)r$$

longitudinal effects



$$U(z)=\int_z^{\infty}\lambda(z')w_{\|}(z'-z)dz' \qquad E\Rightarrow E+qU(z)$$

1. correlated energy spread
2. if λ(z) depends on the energy (low velocity, dispertion region, …) the longitudinal wake introduces a non linear term in the equation of motion

worsen the beam quality and may produce instability

Beam dynamics and multiparticle codes

The tools to study the effects of wake fields are:

1) **analytical methods – generally the linear theory is used**
2) **simulation codes: 10^5 – 10^6 macro-particles are used to represent the bunch**

- **full 3D codes can be used: each particle is represented by 6 differential equations. Simulations are time consuming due to the convolution of the wake fields with the bunch distribution**
- **simulations reproduce quite well the experimental observations of beam behavior**
- **one important disadvantage: due to the limited number of macro-particles, the numerical noise introduced in the simulations makes it hard to study the thresholds of beam instability**

Vlasov equation

An alternative approach consists in describing the bunch by a continuous distribution function. The differential equation governing this function is the Vlasov equation

$$\frac{\partial}{\partial s}\rho = H\rho \quad \textbf{with} \quad \rho\,|_{s=s_0} = \rho_0$$

The operator H, containing the physical properties of the problem, may be specified by simple differential operators, when describing the bunch evolution through magnetic systems, or by integral operators, when accounting for non linear problems associated with the effects of wake fields on the beam.

E.g. for a 2D case in a magnetic bunch compressor, we have

$$H = \frac{x}{R}\frac{\partial}{\partial z} + \theta\frac{\partial}{\partial x} + \left(-k_\beta^2 x + \frac{\varepsilon}{R}\right)\frac{\partial}{\partial\theta} - \frac{Ne^2}{E_0}\frac{\partial}{\partial\varepsilon}\int_{-\infty}^{\infty} w_{\parallel}(z'-z)\lambda(z')dz'$$

Exponential operators

Formal solution: $\rho = \exp(H\,\delta s)\,\rho_0$

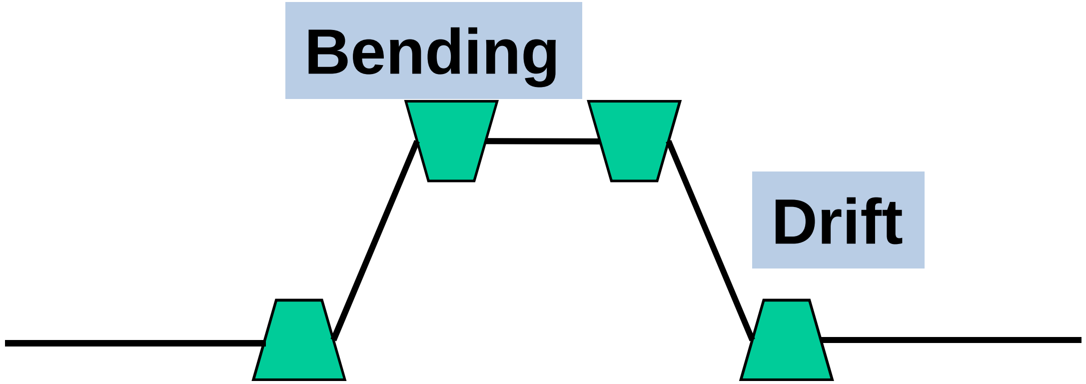
N.B. we are neglecting any contribution due to time ordering corrections, that arises whenever the operator H is explicitly time-dependent and does not commute with itself at different times. With these assumptions we neglect third order terms in the integration step δs. We preserve anyway the symplecticity.

Advantage compared to tracking codes: it gives a very smooth evolution of the beam distribution function that allows to reduce, and in some cases to completely eliminate, the effect of numerical noise.

Disadvantage: 2D and 3D simulations are heavy from the memory point of view. A grid of 100 nodes in each dimension means:
2D → 100^4 nodes → 750MB **3D 100^6 nodes → 7.5 TB**

We use the operator splitting technique to write the explicit solution

1D magnetic compressor



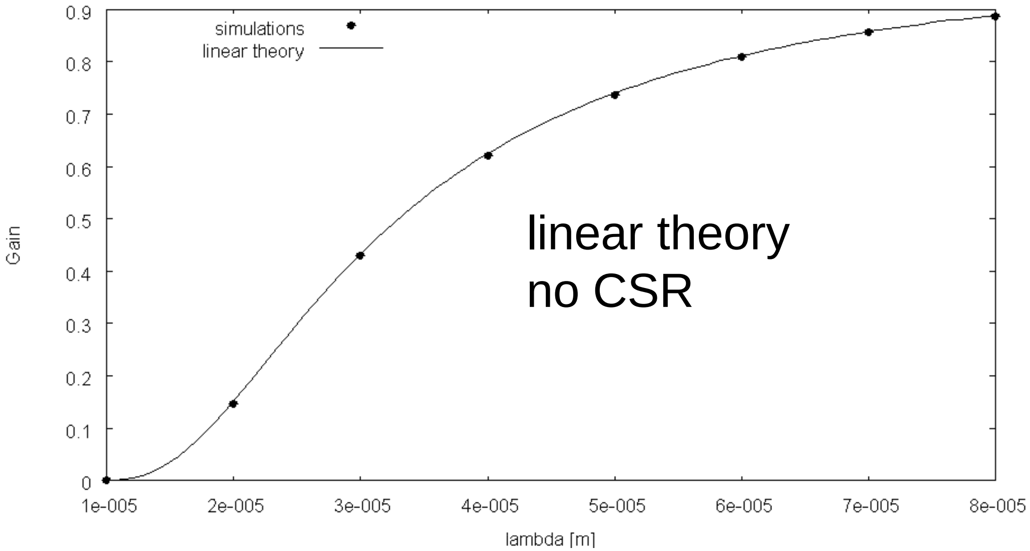
$$\left(\mathrm{CSR}\right)=\frac{1}{4\pi\varepsilon_0}\frac{2}{R^{2/3}}\frac{1}{\left(3z\right)^{4/3}}$$

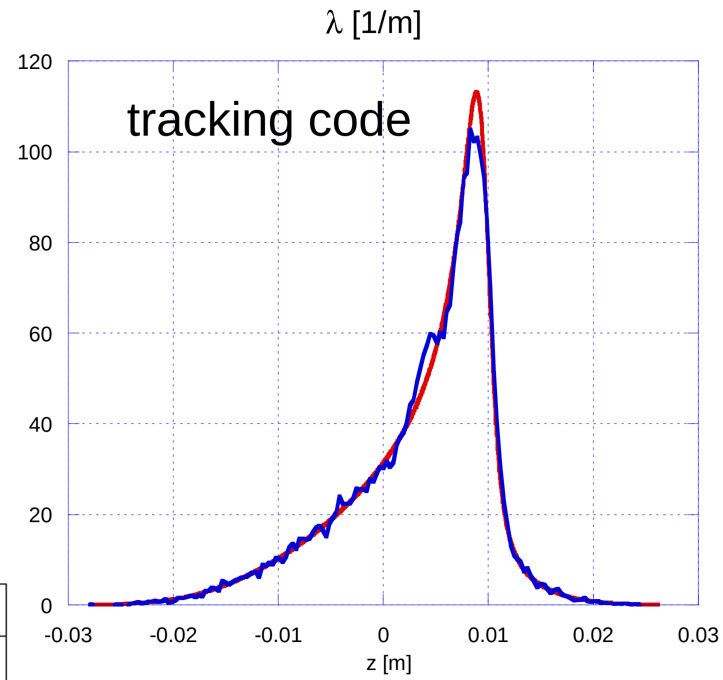
•Initial distribution is sampled on a 2-D uniform Cartesian grid.

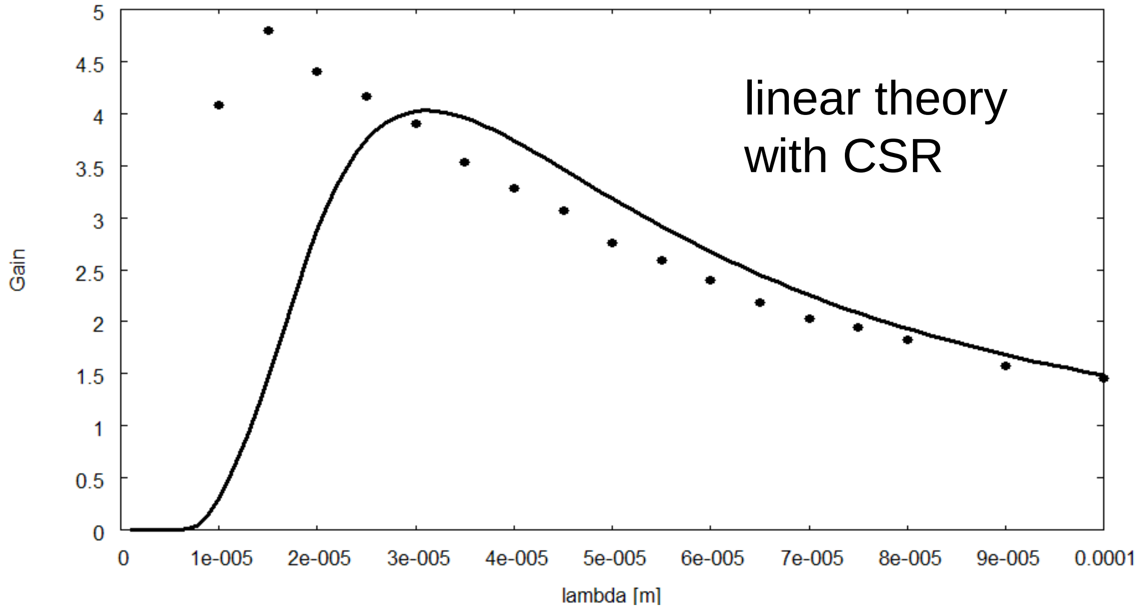
•The distribution is updated step by step along the characteristics (semi-lagrangian method).

•The advanced distribution is obtained at grid points by 2-D interpolation of the initial distribution using fifth-order Lagrange polynomials.

1D magnetic compressor: comparisons





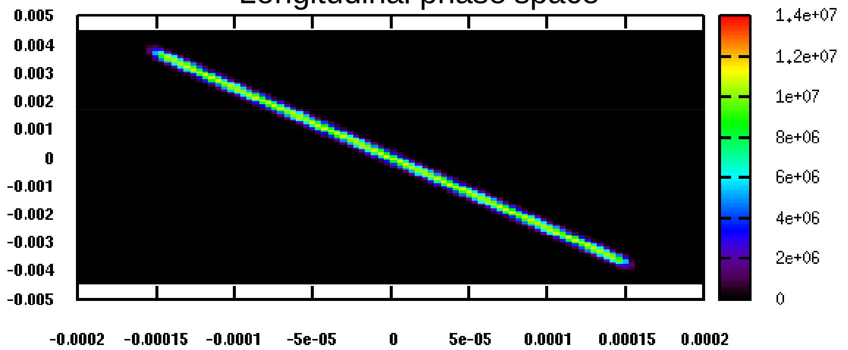


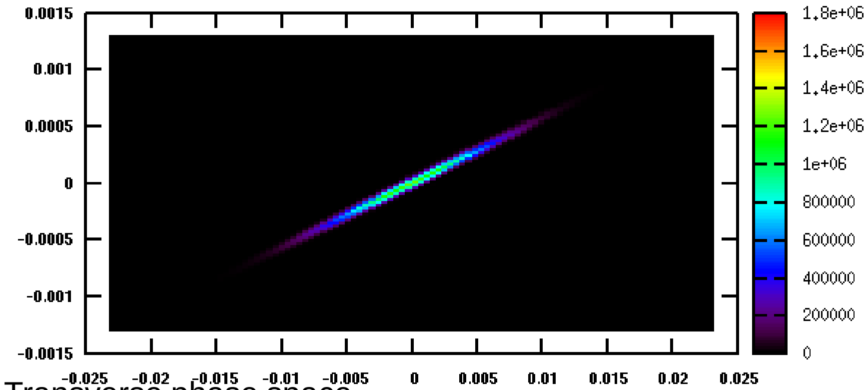
2D magnetic compressor

The work is in progress, the grid must be sufficiently dense

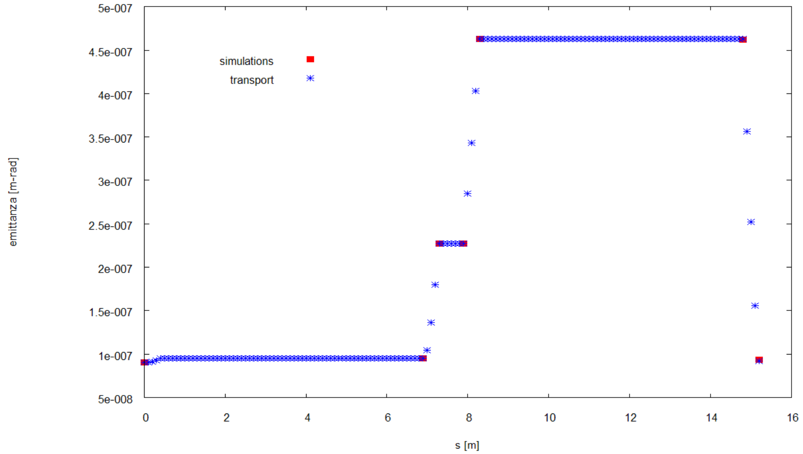
The longitudinal and transverse phase spaces are highly correlated, most of the matrix is not necessary.

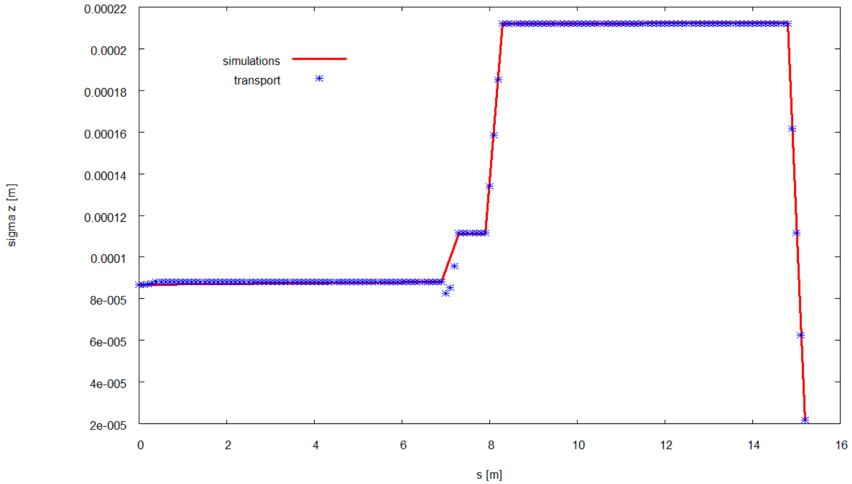
Longitudinal phase space





Transverse phase space





Storage Rings

Fokker Planck equation

$$\rho(z',\varepsilon';\delta s') = \exp\left(\delta s'\right)\exp\left(\frac{\delta s}{2}H\right)\,\exp\left[\delta s'\left(\varepsilon'\frac{\partial}{\partial\varepsilon'}+\frac{\partial^2}{\partial\varepsilon'^2}\right)\right]\exp\left(\frac{\delta s}{2}H\right)\rho_0$$

The difference with respect to the previous analysis is the presence of a new exponential operator containing second order derivatives. It can be written as

$$\exp\left[\delta s'\left(\varepsilon'\frac{\partial}{\partial\varepsilon'}+\frac{\partial^2}{\partial\varepsilon'^2}\right)\right]\rho(z',\varepsilon';0) = \frac{\exp\left(-\delta s'\right)}{2\sqrt{\pi}\left[1-\exp\left(-\delta s'\right)\right]}\int_{-\infty}^{\infty}\exp\left[\frac{-(\varepsilon'-x)^2}{4\left[1-\exp\left(-\delta s'\right)\right]}\right]\rho(z',\exp\left(-\delta s'\right)x;0)dx$$

The technique is interesting for the study of the microwave instability threshold.

Conclusions

- **A Vlasov solver that uses the exponential operator technique has been written for a magnetic bunch compressor in 1D case and work is in progress for the 2D code.**

- **It includes the wake field effects and simulates the evolution of beam distribution with much lower numerical noise level compared to particle tracking codes.**

- **It can be considered as an alternative approach to the study of non linear dynamics in particle accelerators, especially useful in the study of the onset of instabilities and in the determination of their thresholds.**