PROMETEO
a one dimensional code for FEL design

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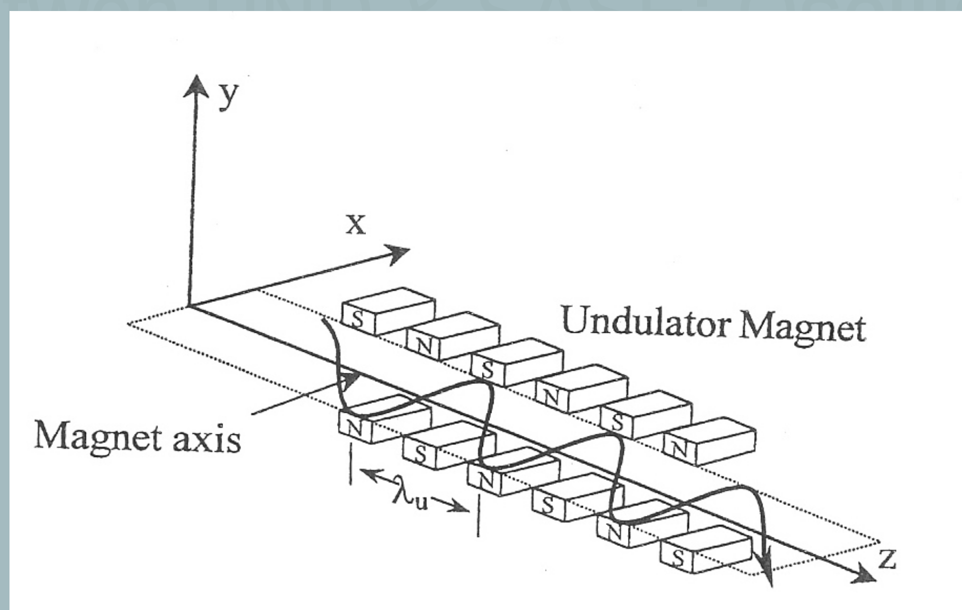
PROMETEO is a 1-D multiparticle code aimed at simulating Free Electron Laser processes: electron bunches travelling within a device (undulator) with periodical magnetic field interact with their own co-propagating radiation.

The code can take into account different devices: oscillator, optical klystron, tandem oscillator. It includes the effects of beam transverse emittance, noise in seed and in electron initial distribution, pulse propagation and diffraction effects. This code accounts for the evolution of the fundamental harmonic and for the coherent generation of higher- order harmonics. It can treat both linear and helical undulators.



A periodic magnetic field in the undulator may be

$$\vec{B} = -\vec{\nabla}\chi \qquad \chi = -\frac{B_0}{k_y}\cosh(k_x x)\sinh(k_y y)\sin(k_u z)$$

$$k_x^2 + k_y^2 = k_u^2 \qquad k_u = \frac{2\pi}{\lambda_u}$$

$$k_x = k_y = \frac{k_u}{\sqrt{2}} \qquad \textit{parabolic pole faces}$$

N *number of undulator periods*

A foundamental undulator parameter is

$$K = \frac{eB_0\lambda_u}{2\pi m_e c^2} \quad undulator \;\; parameter \;\; (of \; the \; order \; of \; unity)$$

The dynamic of the system of an e-beam travelling in an undulator, along the longitudinal z-direction interacting with an e.m. wave is specified by the Lorentz's equations of motion (Gaussian system)

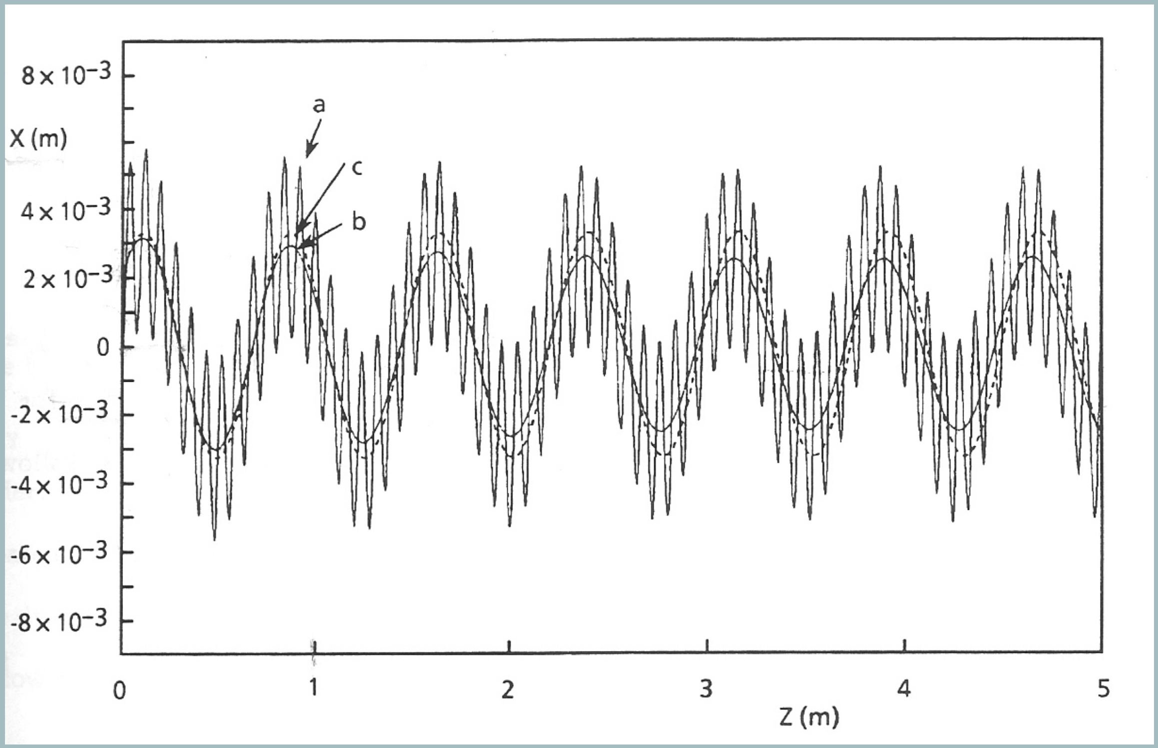
$$m_e \frac{d}{dt}(\gamma \vec{v}) = -e\left(\vec{E}_s + \frac{1}{c}\vec{v} \times \vec{B} \right)$$

$$\frac{d}{dt}\gamma = -\frac{e}{m_e c^2}\vec{E}_s \cdot \vec{v}$$

The electron motion is composed of two terms $\vec{r} = \vec{r}_0 + \vec{r}_1$

$\vec{r}_0$ slowly varying (betatron motion) with period of several λ_u

$\vec{r}_1$ rapidly oscillating (wiggler motion) with period λ_u



a) total motion

b) betatron motion

$$\gamma = 16, \quad \lambda_u = 8\ cm, \quad K = 3.13 \qquad \lambda_\beta = \frac{2\pi}{k_\beta} \approx 0.8\ m$$

Neglecting interaction with e.m. field (constant γ , transport simulation) considering $\frac{K}{\gamma}, \; k_u x, \; k_u y$ as $0(\varepsilon)$ and defining $\vec{R} = k_u \vec{r}$ we have

$$\frac{d^2}{ds^2}(X_0, Y_0) = -k_\beta^2 \left[(X_0, Y_0) + 0\left(\varepsilon^3\right) \right] \qquad s = ct \qquad k_\beta = \frac{\pi K}{\lambda_u \gamma}$$

$$\frac{d}{ds} Z_0 = k_u \beta_0 \left[1 + 0\left(\varepsilon^6\right) \right]$$

$$X_1 = -\frac{1}{\beta_0} \frac{K}{\gamma} \left[\sin(\alpha) + 0\left(\varepsilon^3\right) \right] \qquad \alpha = k_u v_0 t \qquad \beta_0 = \frac{v_0}{c}$$

$$Y_1 = 0\left(\varepsilon^3\right)$$

$$Z_1 = -\frac{1}{8\beta_0^2} \left(\frac{K}{\gamma} \right)^2 \sin(2\alpha) + \frac{1}{k_u \beta_0^2} \frac{K}{\gamma} \left(\frac{d}{ds} X_0 \right) \sin(\alpha) + 0\left(\varepsilon^4\right)$$

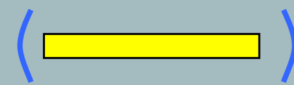
v_0 *constant*

PROMETEO code can take into account, thanks to a very flexible input, different types of devices: single-pass amplifier, optical klystron oscillator, tandem oscillator, biharmonic undulator, storage ring, sub-harmonics amplifiers, etc.).

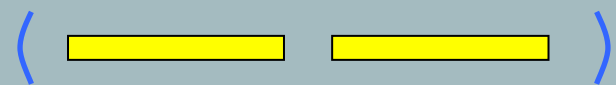
A sequence with up to three undulators with different parameters can be taken into account.

Different devices can be obtained through mirror insertion:

oscillator



optical klystron (OK)



tandem FEL

oscillator +
sub-harmonic amplifier

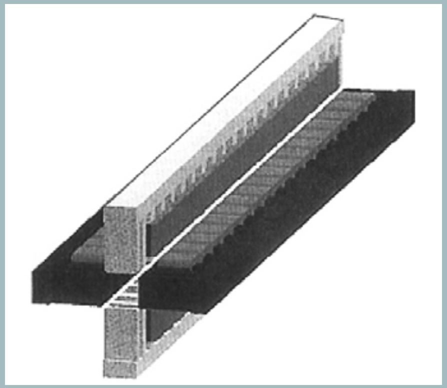
OK + amplifier

... and so on

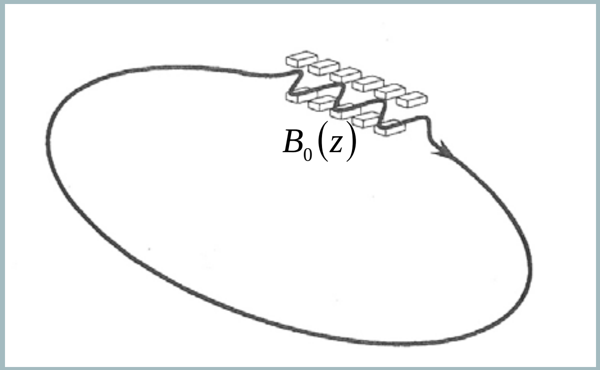
Special configurations and feasibilities

- Segmented undulators

- Biharmonic linear undulator



- Insertion device in a straight section of a Storage Ring



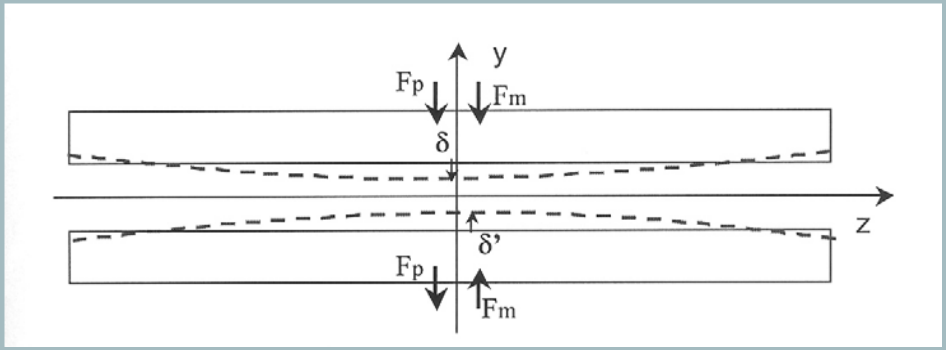
- B_0 tapering ➔ $B_0(z)$

A selected $B_0(z)$ profile is assigned

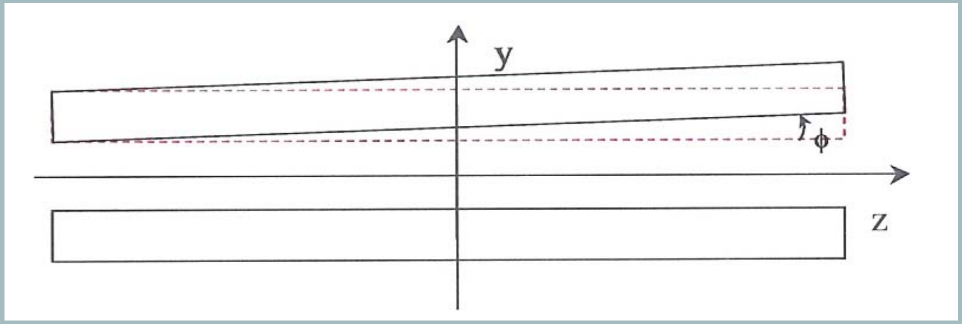
$B_0(z)$ is derived from resonant condition

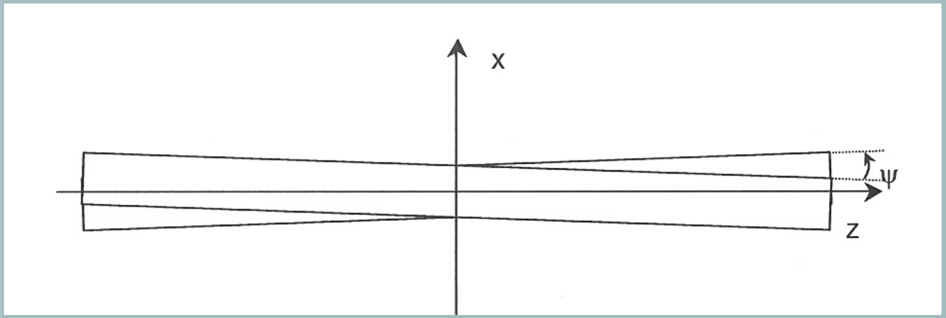
Undulator defects

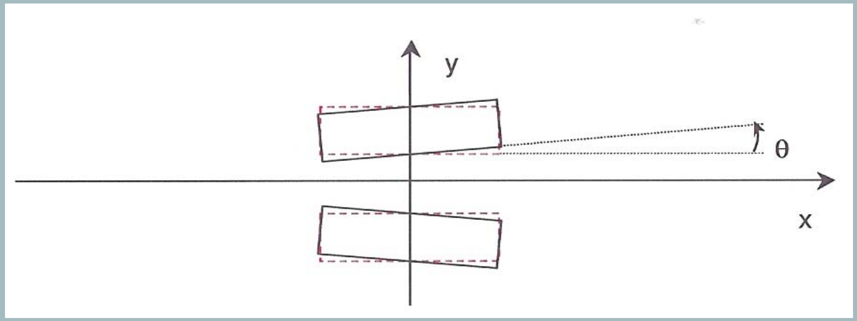
- Deflection of the up and down magnet surfaces (dotted lines).
  δ maximum deflection,
  F_m magnetic force,
  F_p weight force



- Misalignments of the sections in a segmented undulator







- Small random variations $\Delta\lambda_u$ and ΔB_0

e-beam model

e-beam energy distribution

$$\Phi(\varepsilon) = \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} \exp\left(-\frac{\varepsilon^2}{2\sigma_\varepsilon^2} \right) \quad \textit{relative energy distribution}$$

$$\varepsilon = \frac{\gamma - \gamma_0}{\gamma_0}, \qquad \sigma_\varepsilon \quad \textit{relative energy and energy spread}$$

$$\mu_\varepsilon = 4N\sigma_\varepsilon \qquad \textit{energy distribution inhomogeneous broadening parameter}$$

e-bunch longitudinal profile

$$f(t) = \exp\left[-\frac{1}{2}\left(\frac{t}{\sigma_b} \right)^2 \right]$$

$$\sigma_b \qquad e-\textit{bunch length (r.m.s. standard deviation)}$$

$$\Delta = N\lambda_0 \quad \textit{slippage length}$$

$$\mu_c = \frac{\Delta}{\sigma_b} \quad \textit{longitudinal mode coupling parameter}$$

e-beam phase space distribution

(no coupling between x, y planes)

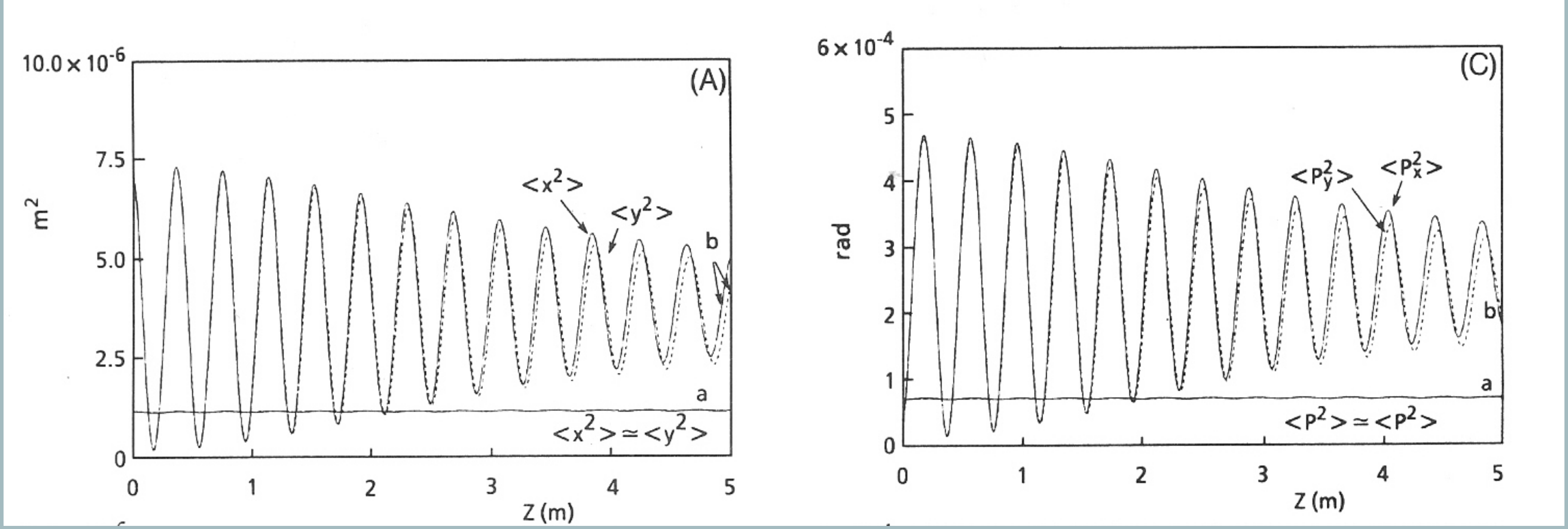
$$F(x,x',y,y') = W(x,x')W(y,y') \quad \textit{distribution function}$$

$$W(\eta,\eta') = \frac{1}{2\pi\varepsilon_\eta}\exp\left[-\frac{1}{2\varepsilon_\eta}\left(\beta_\eta \eta'^2 + 2\alpha_\eta \eta\eta' + \gamma_\eta \eta^2\right)\right]$$

ε_η — *emittance in the* (η,η') *plane*

$\alpha_\eta, \beta_\eta, \gamma_\eta$ — *Twiss coefficients with* $\beta_\eta \gamma_\eta - \alpha_\eta^2 = 1$

matched beam $\alpha_\eta = 0, \ \beta_\eta = k_\beta^{-1}$



beam envelope vs z.

a) initial matched beam ;   b) nonmatched beam

1-D formulation

The on axis magnetic field in a linearly polarized undulator is

$$\vec{B}_u \equiv \left[0,\ B_0 \sin(k_u z),\ 0\right]$$

Linear light polarization in the electron orbit plane (x, z) is generated

$$\vec{E}_s \equiv \left[E_s \cos\psi_s,\ 0,\ 0\right]$$

$$\vec{B}_s \equiv \left[0,\ E_s \cos\psi_s,\ 0\right]$$

$$\psi_s = k_s z - \omega t + \phi_s$$

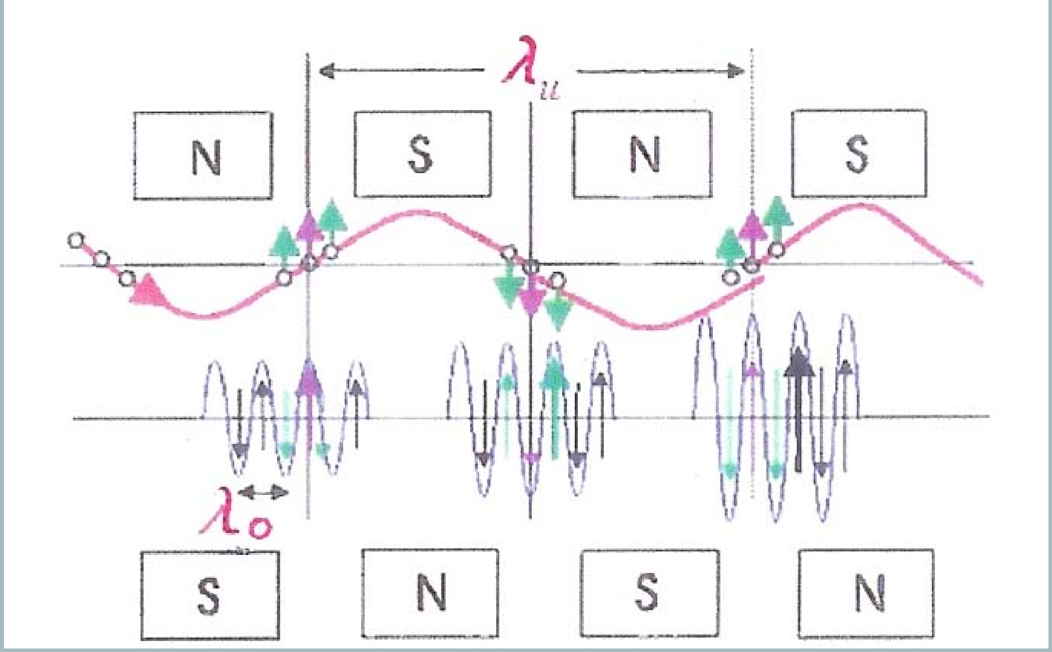
$$\lambda_s\ , \quad k_s = \frac{2\pi}{\lambda_s}, \quad \omega = ck_s$$

laser wave length, wave number and frequency

Radiation emitted in the forward direction near the resonant wavelength λ_0

$$\lambda_0 = \frac{\lambda_u}{2\gamma^2}(1+\frac{K^2}{2})$$

$$\gamma = \frac{E}{m_e c^2} \qquad \textit{electron relativistic factor, E electron energy}$$



Resonance condition: co-propagating radiation overtakes the electrons in one undulator period by a slippage length equal to λ_0. Radiation exchange energy with the electrons over many undulator periods resulting in a gain of the its energy at the expense of the mean beam kinetic energy.

1-D FEL equations – continuous e-beam approximation $\lambda_s << \sigma_\beta$

e.m. field feels electrons with the same current density J .

From Lorentz force and Maxwell equations, averaging over an undulator period:

$$\frac{d\gamma}{dz} = \frac{1}{\sqrt{2}}\frac{K}{\gamma} e_s f_b \cos\psi$$

$$\frac{d\zeta}{dz} = k_u - \frac{k_u + k_s}{2\gamma^2}\left[1 + \frac{K^2}{2} + \left(\frac{e_s}{k_s}\right)^2 - \sqrt{2}K\frac{e_s}{k_s} f_b \sin\psi\right]$$

$$\frac{d e_s}{dz} = -2\pi\frac{e}{m_e c^3}\left|J\right|\frac{K}{\sqrt{2}} f_b \left\langle\frac{\cos\psi}{\gamma}\right\rangle$$

$$e_s\frac{d\phi_s}{dz} = 2\pi\frac{e}{m_e c^3}\left|J\right|\left[\frac{K}{\sqrt{2}} f_b\left\langle\frac{\sin\psi}{\gamma}\right\rangle - \frac{e_s}{k_s}\left\langle\frac{1}{\gamma}\right\rangle\right]$$

$$\psi = \zeta + \phi_s$$

$$\zeta = (k_u + k_s)\int_0^t \bar{v}_z(t')dt' - \omega t$$ *dimensionless electron phase*

$\bar{v}_z$ *denotes average over the undulator period*

$\langle \quad \rangle$ *indicates average over electrons in a ψ period*

e_s *normalized electric field amplitude* $[l^{-1}]$

f_b *Bessel function factor*

$$e_s = \frac{eE_s}{\sqrt{2} m_e c^2}, \qquad f_b = J_0(\xi) - J_1(\xi)$$ $J_{0,1}$ *cylindrical Bessel function*

$$\xi = \frac{1}{4}K^2\left(1 + \frac{K^2}{2}\right)^{-1}$$

J *electron beam current density*

ζ is the electron’s phase relative to the e.m. wave.

Foundamental parameters characterizing the power evolution are:

$$g_0 = \frac{16\pi}{\gamma}\frac{|J|}{I_0}\lambda_0 L N^2 \xi f_b^2 \qquad \textit{small signal gain FEL coefficient}$$

$$= 4\pi \frac{|J|}{I_0}\left(\frac{N}{\gamma}\right)^3 \left(\lambda_u \frac{K}{\sqrt{2}} f_b\right)^2$$

$$I_S = \frac{c}{8\pi}\left(\frac{m_e c^2}{e}\right)^2 \left(\frac{\gamma}{N}\right)^4 \left(\lambda_u \frac{K}{\sqrt{2}} f_b\right)^{-2} \qquad \textit{saturation intensity}$$

$$\rho = \frac{1}{4\pi}\left[\frac{\pi g_0}{N^3}\right]^{\frac{1}{3}} \qquad \textit{Pierce parameter}$$

The gain is a simple function of g_0

I_S is the field intensity halving the small signal gain.

$$P_E = E \cdot \frac{|J|}{e} = \frac{m_e c^2}{e}\gamma |J| = 2 N g_0 I_S \quad e-beam \ \ power \ \ density$$

$$L_g = \frac{\lambda_u}{4\pi\sqrt{3}\rho} \qquad gain \ length$$

$$P_F \cong \rho P_E \quad saturated \ \ power \ (maximum \ \ achievable \ \ power)$$

$$Z_F \cong L_g \ln\left(\frac{9 P_F}{P_0}\right) \quad saturation \ \ length$$

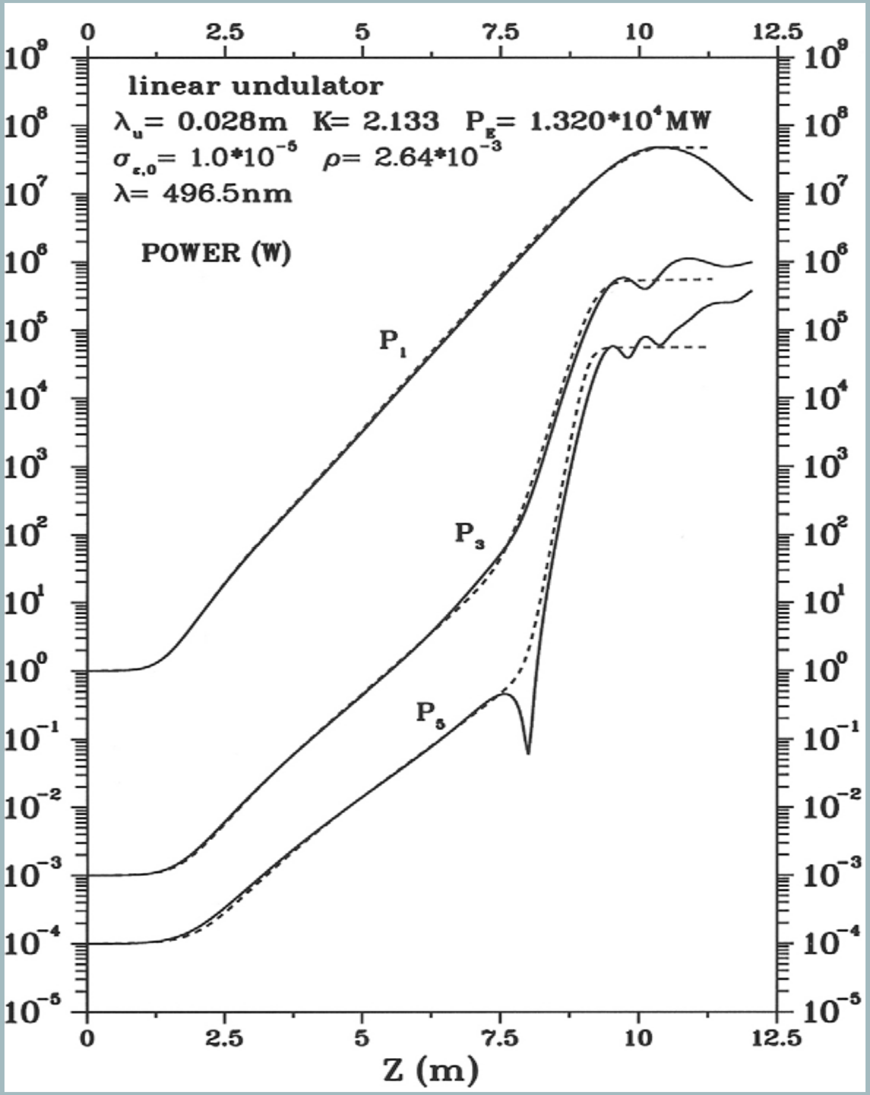
$$P_0 \quad input \ \ seed \ \ power$$

Power growth of the first three harmonics in a high gain FEL amplifier.

Continuous line: 1-D simulation, dashed lines: theoretical formulas from

G. Dattoli, P.L.Ottaviani,

S. Pagnutti RT/2007/40/FIM



Radiation parameters

$$\lambda_0 = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right),$$

$$\omega_0 = 2\pi c \big/ \lambda_0 \qquad \textit{resonant wavelength and frequency (central emission)}$$

$$\lambda_s = \lambda_0\left(1 - \nu \big/ 2\pi N\right)^{-1}; \quad \nu = L\,\frac{d\zeta}{dz} = 2\pi N\,\frac{\omega_0 - \omega}{\omega_0} \qquad \textit{detuning parameter}$$

$$f(\nu) = \left[\mathrm{sinc}\left(\nu \big/ 2\right)\right]^2 \qquad \textit{spontaneous emission line shape}$$

$$I = \frac{c}{4\pi}\left(\frac{m_e c^2}{e}\right)^2 e_s{}^2 \qquad \textit{intensity}$$

Macroparticle electron variables (γ_i, ς_i)

N_γ initial values γ_i from a Gaussian distribution around γ

N_ς initial values ζ_i from equipartition of the interval $]-\pi, \pi[$

Shot noise microbunching are taken into account with small variations of ζ_i following the procedure of

- W. M. Fawley, Phys. Rev. Special Topics- Acc. and Beams 5 (2002) 70701
- C. Penman, B. W. J. McNeil, Optics Comm. 90 (1992) 82

e.m. field variables e_s , ϕ_s

$e_s(z=0)$ from an initial seed

$2(N_\gamma \cdot N_\varsigma)+2$ coupled differential equations

z integration by Adams-Bashforth-Moulton predictor corrector iterative method:

predictor step Adams-Bashforth of order 2, corrector step Adams-Moulton of order 3

A set of similar equations holds including sub-harmonic generation with amplitude and phase e_{sn} , ϕ_{sn}

Approximate treatment of emittance

In pure 1-D treatment the transverse electron velocity averaged over

an undulator period is:

$$\overline{\beta}_{\perp}^{2} = \frac{1}{\gamma^{2}}\left[\frac{K^{2}}{2} + \left(\frac{e_{s}}{k_{s}}\right)^{2} - \sqrt{2}K\frac{e_{s}}{k_{s}} f_{b} \sin\psi\right]$$

entering in $\frac{d\zeta}{dz}$ equation.

In 3-D space a macroparticle may be characterized by ($x,\ p_x, y, p_y, \gamma, \zeta$).
The transverse velocity, assuming as above a plane wave for the e.m. field, is the sum of the above term plus a contribution H which in a transport treatment is a constant of motion (except for term $0(\varepsilon^{6})$).

To account for this contribution we assume (γ_i, ζ_i, H_i) as macroparticle where H_i is a discretized value of H with an assigned weight determined by all the values of ($x,\ p_x, y, p_y$) giving a contribution around H_i .

The equation for ζ becomes

$$\frac{d\zeta}{dz} = k_u - \frac{k_u + k_s}{2\gamma^2}\left[1 + \frac{K^2}{2} + \gamma^2 H + \left(\frac{e_s}{k_s}\right)^2 - \sqrt{2}K\frac{e_s}{k_s} f_b \sin\psi\right]$$

For small γ variations, H may be considered constant .

Diffraction correction

The longitudinal mode transverse size may be accounted for by introducing

$$\mu_\eta^D = \frac{\lambda_0 \lambda_u}{(4\pi)^2 \beta_\eta \varepsilon_\eta \rho} \qquad \textit{diffraction correction parameter}^{(*)}$$

(*) G.Dattoli et al. J. Appl. Phys. 95 (2004) 3206

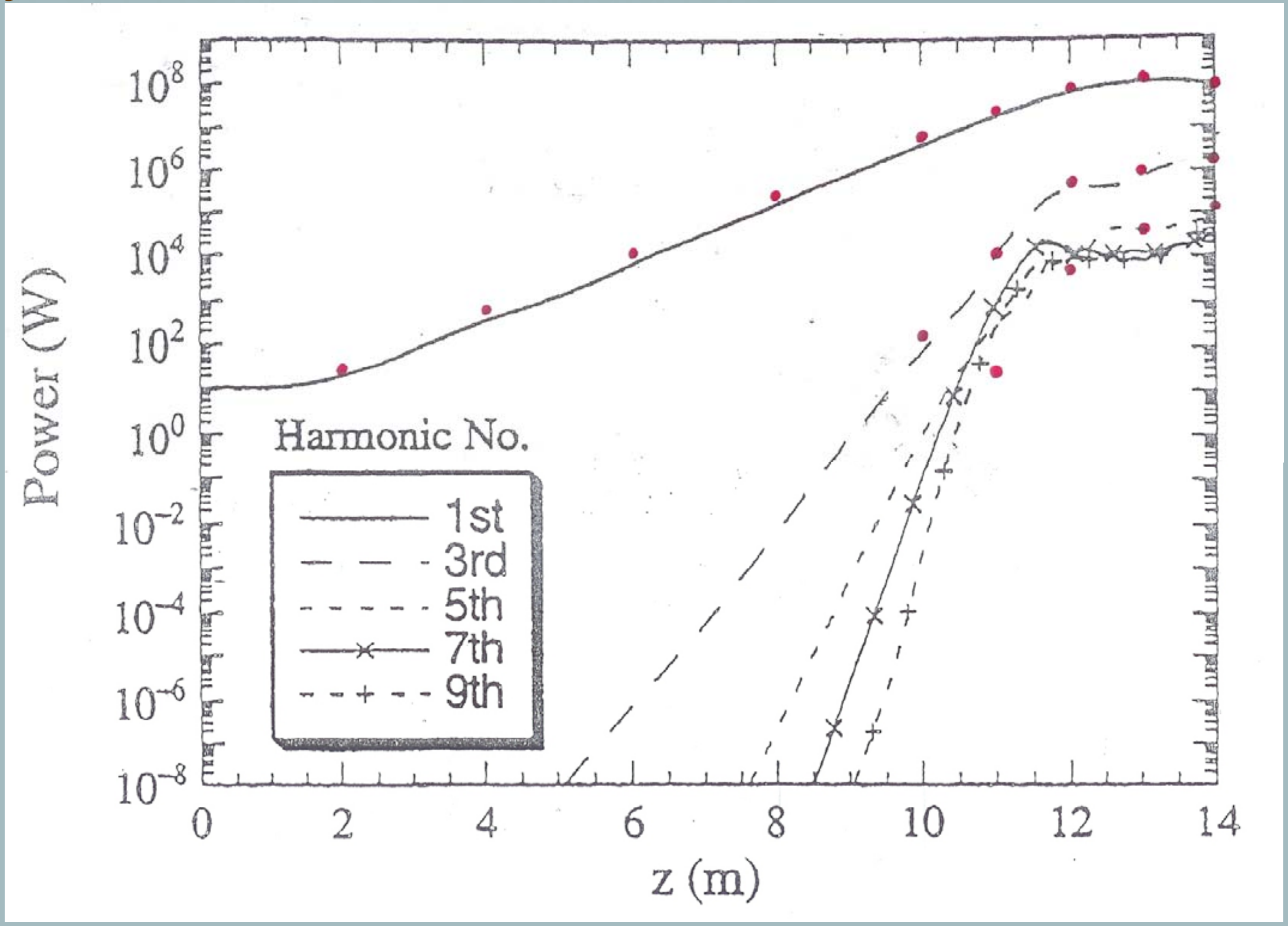
which has the effect of reducing the Pierce parameter.

$$\rho_1^D = F\left(\mu_x^D, \mu_y^D\right)\rho$$

$$F\left(\mu_x^D, \mu_y^D\right) = \left[\left(1+\mu_x^D\right)\left(1+\mu_y^D\right)\right]^{-1/6}$$

In a 1-D approximation this is equivalent to a reduction of the density current J

Validity of PROMETEO 1-D simulations



Comparison of PROMETEO and MEDUSA simulations at $\lambda \approx 518\ nm$

MEDUSA results from Fig.2 in H.P.Freund, S.G.Biedron, S.V. Milton, IEEE J. Quantum Electron. 36 (2000) 275；red dots: PROMETEO.

- The optimal wavelength λ_s is chosen in order to minimize the gain length in the exponential part of power growth. The same is done in most of 3-D codes (GENESIS, GINGER, MEDUSA, TDA3D). The saturation power is a sensitive function of wavelength within the gain band and small differences in the choices for the wavelength can result in relatively large variations in saturated power

- The saturation length Z_F is less affected by this uncertainty.

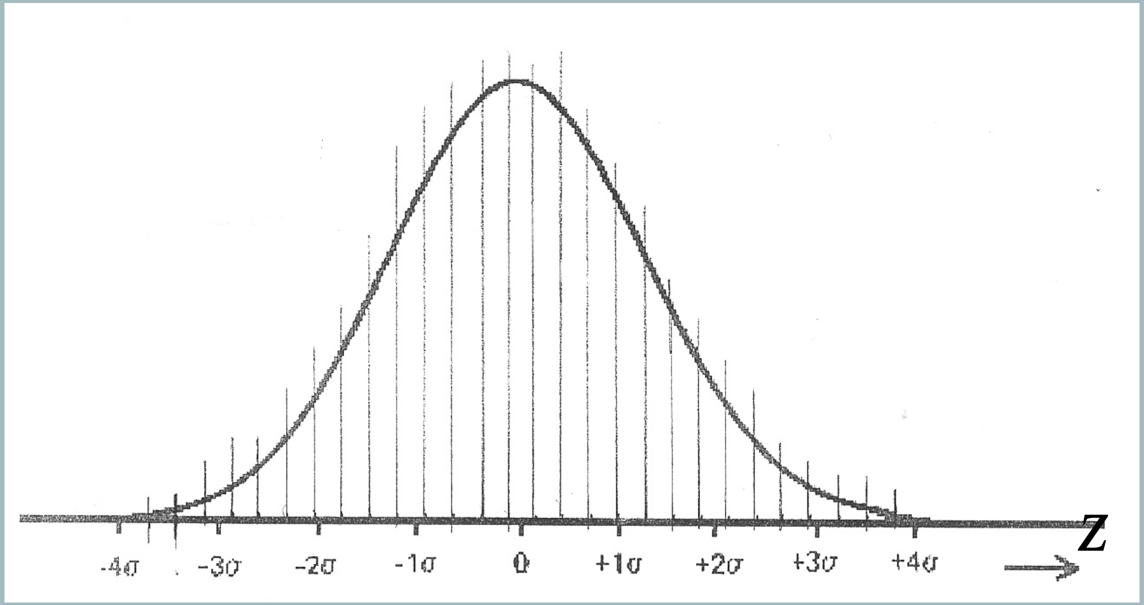
The following table shows the results of different codes for the same set of simulation parameters.

S. G. Biedron et al., Nucl. Instrum. Methods A 445 (2000) 110.
S. G. Biedron et al., Phys. Rev. Special Topics-Acc. and Beams 5 (2002) 30701

Results comparison

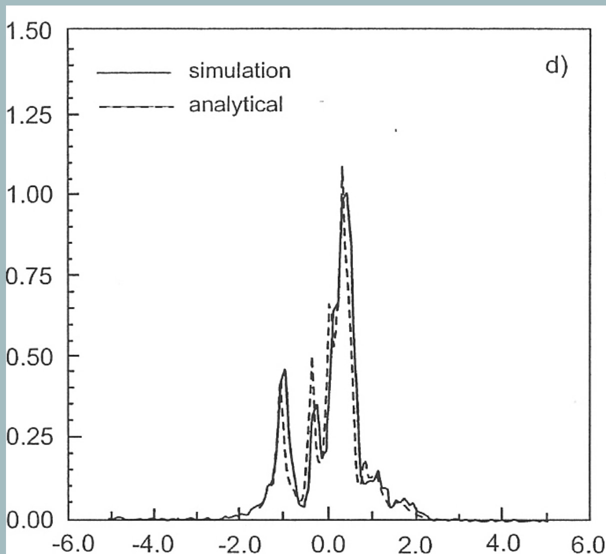
Code	Optimum λ_s (nm)	L_{SAT}	P_{SAT}
GENESIS	517.78	15.5	69
GINGER	519.0	15.	103
MEDUSA	518.82	14.	109
TDA3D	517.78	15.4	69
PROMETEO	518.07	14.7	121.3

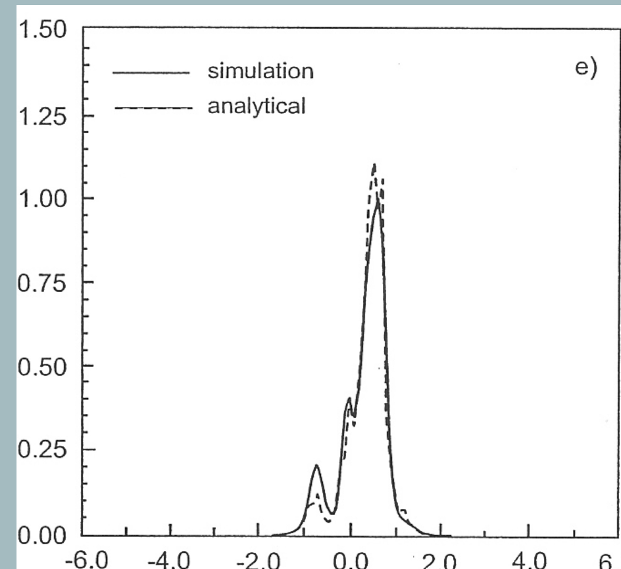
Pulse Propagation

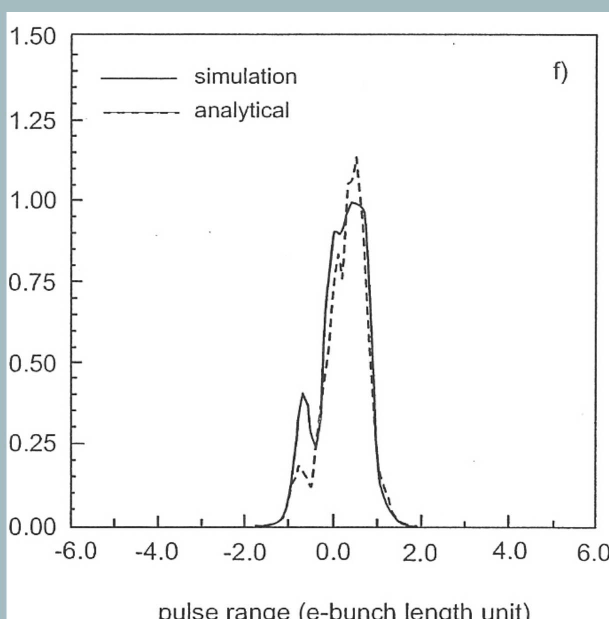


The e-bunch is discretized in n_s slices of length of some λ_0: $d_s = n_\lambda \lambda_0$.
At every slice a set of (γ_i, ζ_i) is assigned (with possibile random noise and initial seed) feeling a different density current.
The e.m. wave overcomes electrons of one λ_0 every undulator period; so to overcome a full slice, n_λ periods of the undulator are necessary. For n_λ periods and for every slice FEL equations are solved. After, the e.m. field generated by slice *i-1* is shifted to slice *i* in the forward direction.

An example of pulse evolution in amplifier







Linear undulator, $E = 156\ Mev,\quad \lambda_u = 3.0\ cm,$

$K = 1.99,\quad \rho = 5.17 \cdot 10^{-3},\quad \sigma_{\varepsilon,0} = 10^{-4},$

$\sigma_b = 96\ \mu m \quad \lambda_0 = 477.8\ nm \quad \mu_c = 0.88$

random seed with mean value 10W.

Coordinate 0. refers to the e-beam center .

d) *z =2.10 ;* e) *z = 4.20* f) *z = 5.28*

analytical from G. Dattoli, P.L.Ottaviani, S. Pagnutti

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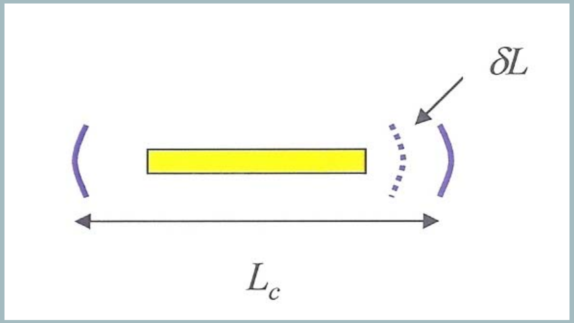
Pulse propagation in oscillator



If time interval between e-bunches is $\frac{2L_c}{c}$ it is assumed a cavity mismatch $\delta L = 0$

The center of mass of the e.m. pulse does not have always the velocity c because

for a length $L_u = N\lambda_u$ it travels together with electrons at velocity v_z

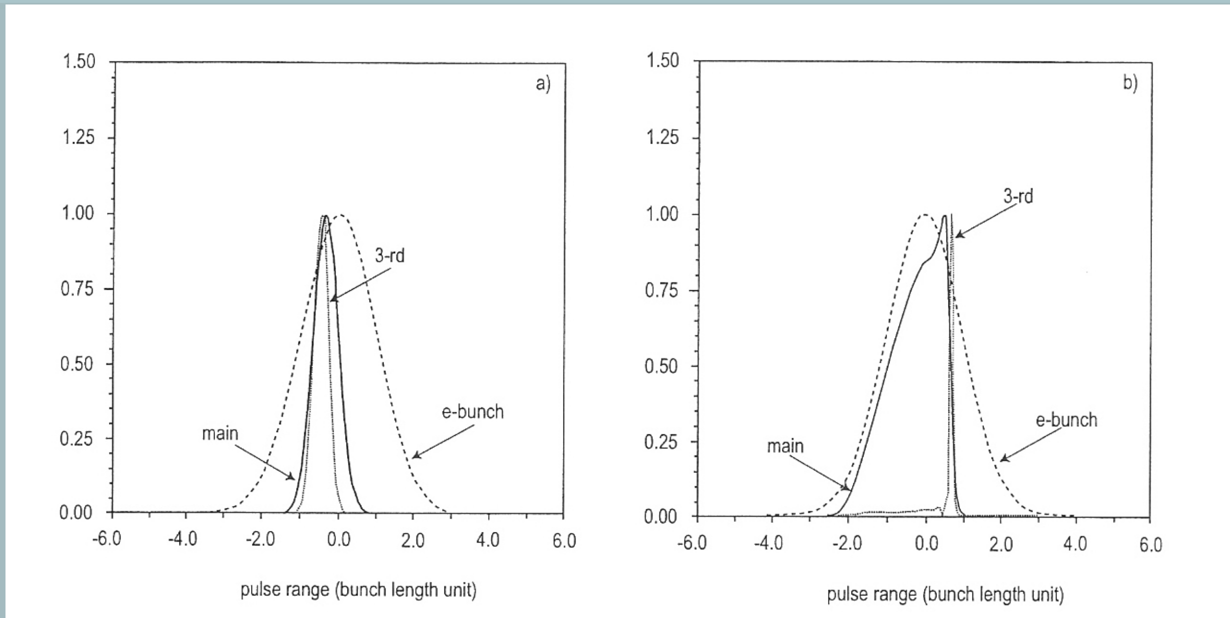
In this case the space retard of the e.m. center of mass is $R = \Delta = N\lambda_0$

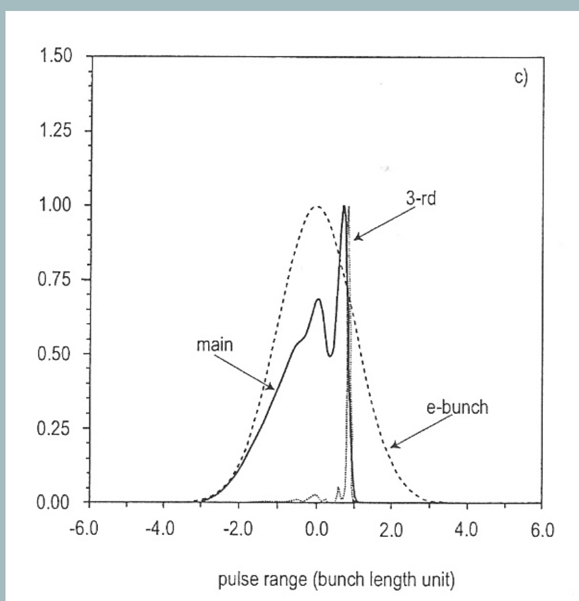
With a δL shift of the mirror:

$\delta L \quad Cavity\ mismatch\ (>0\ for\ cavity\ shortening\ )$

$\theta = \frac{4\delta L}{g_0 \Delta} \quad Cavity\ \det uning\ \ parameter$

$R = \Delta - 2\delta L \quad \mathrm{Re}\,tard$



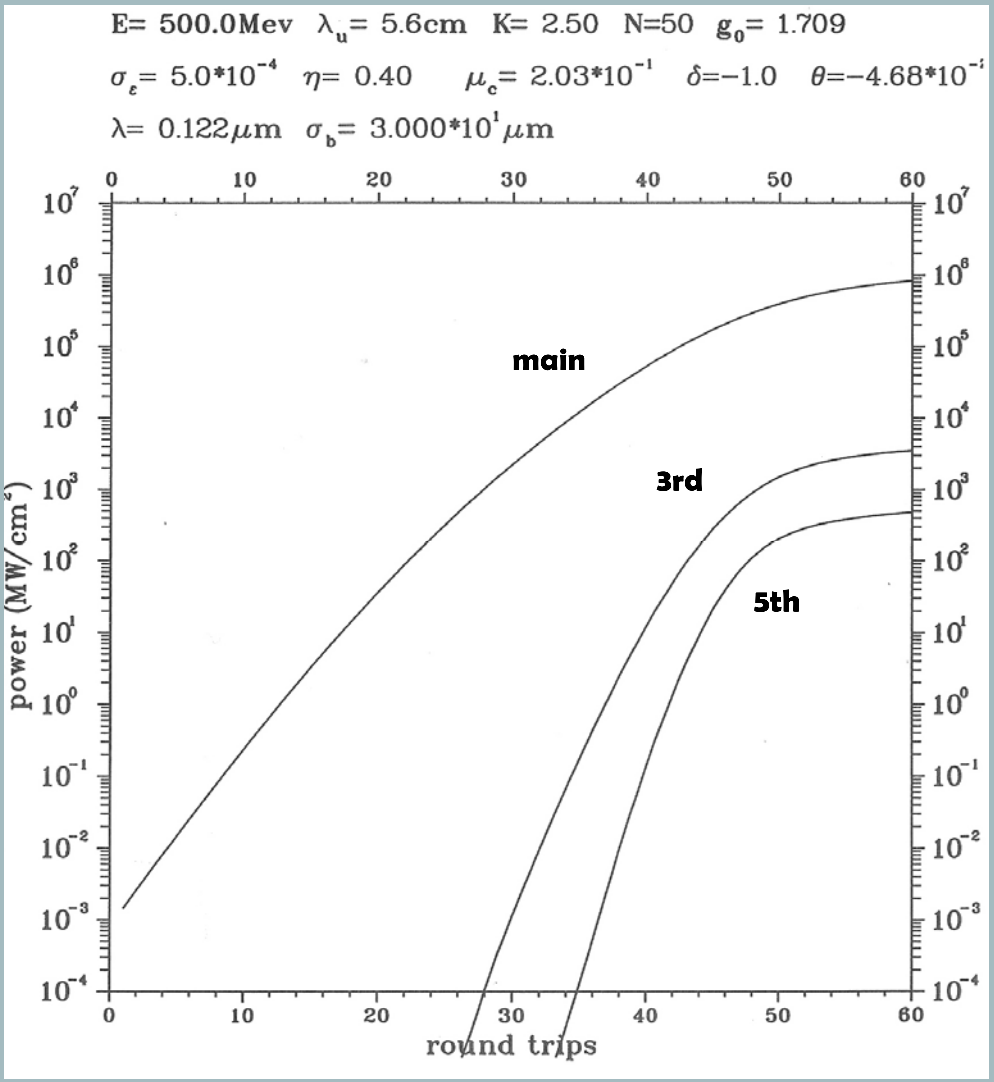


Evolution at different *r.t.* of the fundamental and third harmonic pulse.

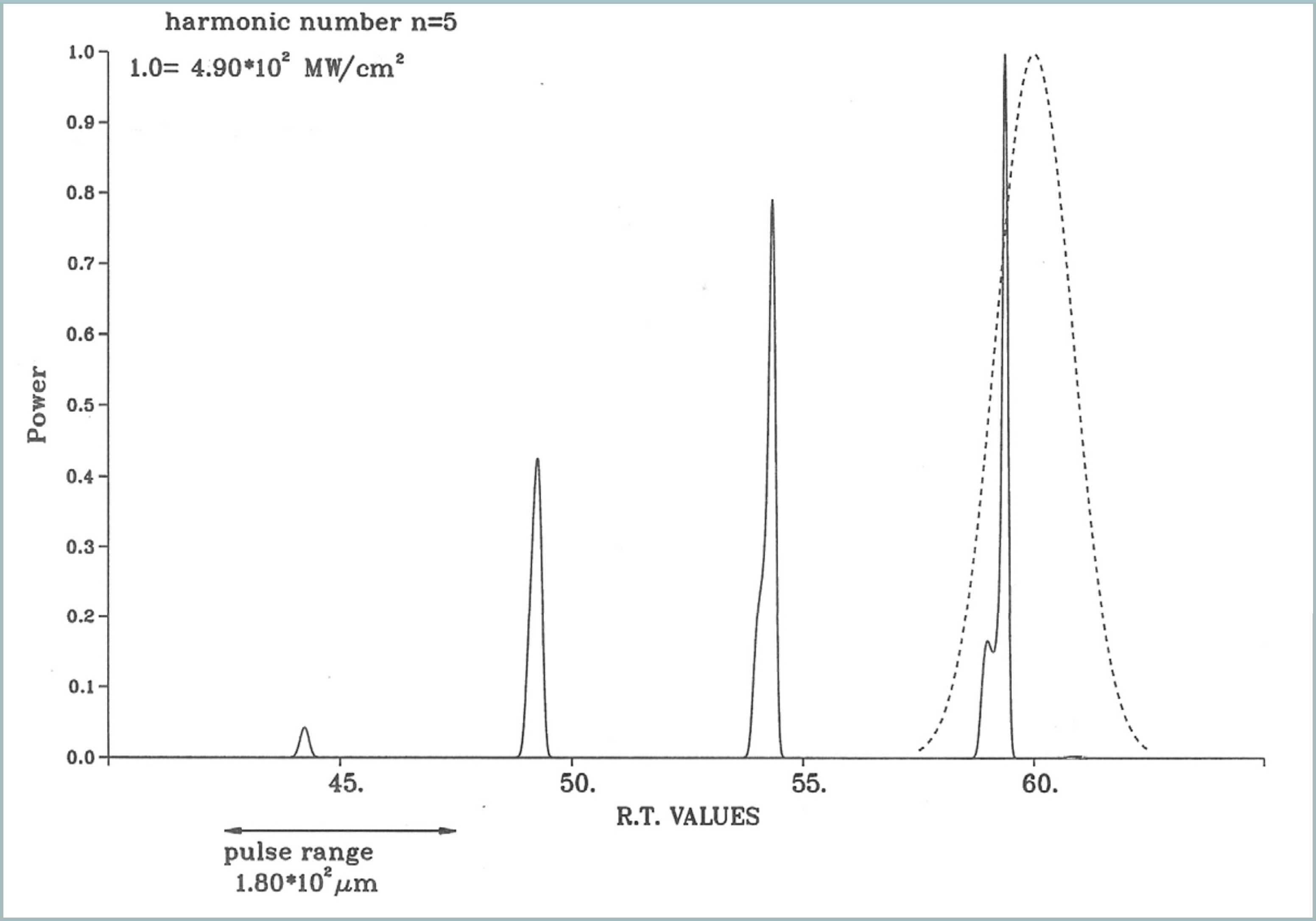
$\lambda_u = 2.8\ cm,\ K = 2.1,\ g_0 = 2,\ N = 50$

$\eta = 6\ \%,\ \mu_c = 0.3,\ \lambda_0 \approx 1\ \mu m,\ \delta L = -1\ \mu m$

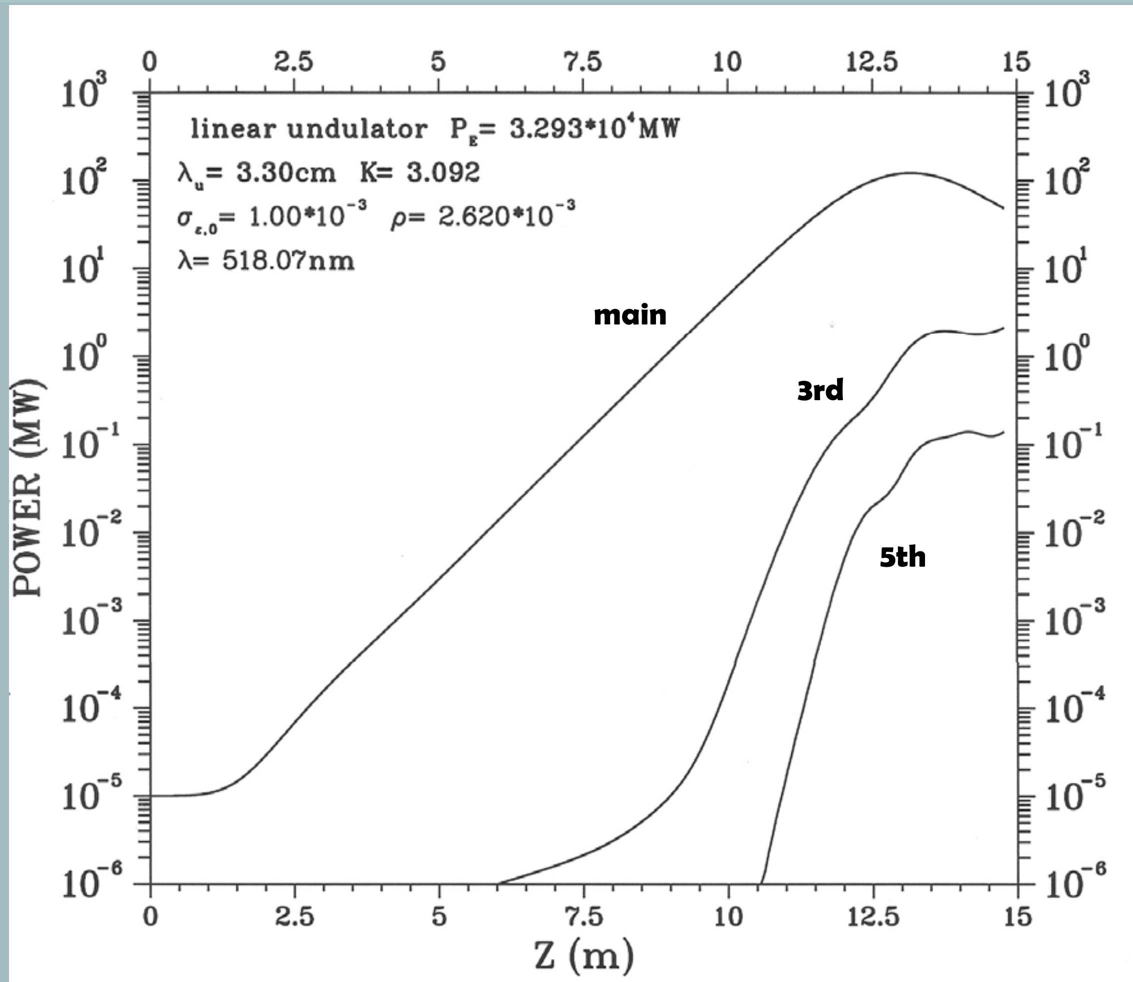
a) *r.t. 12 ,* *b) r.t. 40 ,* *c) r.t.* 120 (saturation)



Oscillator peak power evolution of main, 3rd and 5th harmonics.
Number of slices 750, $d_s = 2\lambda_0$, $\delta = \delta L/\lambda_0 = -1$



Profile evolution of 5th harmonic.



A typical run in amplifier with emittance correction: number of macroparticles $\approx 2.8 \cdot 10^4$, CPU time 76" on an IBM Power4 processor.